Thermal conductivity

Weidemann & Franz observation: (1853)

$$\frac{K}{\sigma} \propto T$$

K: thermal cond. $(W \cdot m^{-1} \cdot K^{-1})$

σ: electrical cond.

Recall: σ has ~~no~~ T dependence

Thermal conductivity: - assume thermal current is carried by conduction $\bar{e}$'s.

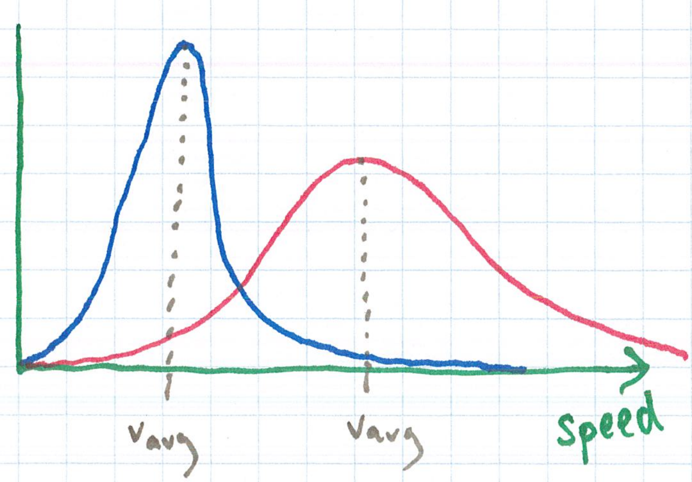
Fourier's Law:

$$\vec{j}_q = -K \vec{\nabla} T$$

$\vec{\nabla} T$: temp. gradient $(K \cdot m^{-1})$

$\vec{j}_q$: thermal current density (or heat flux) $(W \cdot m^{-2})$

Recall: Maxwell-Boltzmann distribution for a gas 3D



Low T

HIGH T

For both $\langle \vec{v} \rangle = 0$

but $v_{avg} \neq 0$

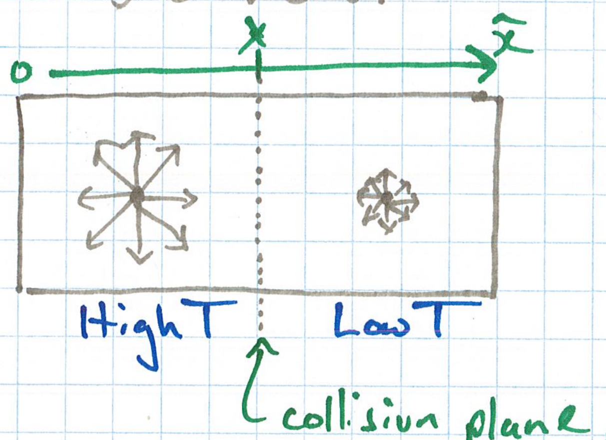
→ after collision $v \propto T$, but direction is random.

Key assumption:

$$\mathcal{E}(T(x))$$

thermal energy of $\bar{e}$ is a function of the temperature @ the position of the $\bar{e}$'s previous collision

→ using this we can approximate the heat transfer in the metal.



- choose collision plane s.t. $\frac{1}{2}$ $\bar{e}$'s come from left / right

High T	Last Collision	Low T
$x - v_H \tau$	x	$x + v_L \tau$ → $\tau \neq \tau(T)$
$\mathcal{E}(T(x - v_H \tau))$	$\mathcal{E}$	$\mathcal{E}(T(x + v_L \tau))$

→ assume slow gradient (∇T small) s.t. $v_H = v_L$

ADD left & right to get thermal current density

$$\bar{j}_q = \frac{\# e^-}{\text{vol.}} \times \begin{matrix} e^- \text{ therm.} \\ \text{energy} \end{matrix} \times e^- \text{ velocity}$$

$$\bar{j}_q = n \varepsilon \bar{v}$$

analogous to electrical.

$$\bar{J} = n q \bar{v}$$

→ at collision plane, $n_H = \frac{n}{2}$

$$n_L = \frac{n}{2}$$

$$\therefore \bar{j}_q = \bar{j}_{q,H} + \bar{j}_{q,L}$$

$$j_q = \frac{1}{2} n v \left[\varepsilon(T(x - v\tau)) - \varepsilon(T(x + v\tau)) \right]$$

→ assume $v\tau$ (MFP) is small. We can expand ε about x.

$$\boxed{j_q = n v^2 \tau \frac{d\varepsilon}{dT} \left(-\frac{dT}{dx} \right)} \quad \text{1D}$$

$$\boxed{j_q = \frac{1}{3} v^2 \tau C_v (-\nabla T)} \quad \text{3D}$$

Energy Expansion: assume $v\tau$ is small

$$\mathcal{E}(T(x-v\tau)) = \mathcal{E}(T(x)) + (x-v\tau-x)\frac{\partial \mathcal{E}}{\partial x} + \dots$$

$$\mathcal{E}(T(x+v\tau)) = \mathcal{E}(T(x)) + (x+v\tau-x)\frac{\partial \mathcal{E}}{\partial x} + \dots$$

$$\therefore \quad \mathcal{E}(T(x-v\tau)) \approx \mathcal{E}(T(x)) - v\tau\frac{\partial \mathcal{E}}{\partial x}$$

$$\mathcal{E}(T(x+v\tau)) \approx \mathcal{E}(T(x)) + v\tau\frac{\partial \mathcal{E}}{\partial x}$$

Thermal current:

$$j_q = \frac{nv}{2}\left[\mathcal{E}(T(x-v\tau)) - \mathcal{E}(T(x+v\tau))\right]$$

$$= \frac{nv}{2}\left[\mathcal{E}(T(x)) - v\tau\frac{\partial \mathcal{E}}{\partial x} - \left(\mathcal{E}(T(x)) - v\tau\frac{\partial \mathcal{E}}{\partial x}\right)\right]$$

$$j_q = -nv^2\tau\frac{\partial \mathcal{E}}{\partial x} = -nv^2\tau\frac{\partial \mathcal{E}}{\partial T}\frac{\partial T}{\partial x}$$

where: $\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle = \frac{1}{3} v^2$

& $c_v = \frac{1}{V}\frac{dE}{dT} = \frac{N}{V}\frac{d\mathcal{E}}{dT} = n\frac{d\mathcal{E}}{dT}$

$\therefore E = U$
total internal energy
$= N\mathcal{E}$
$\# e^-$ nrg per e^-

$$\boxed{K = \frac{1}{3} v^2 \tau c_v}$$

Thermal conductivity

Wiedemann & Franz:

$$\frac{K}{\sigma} = \frac{m v^2 \tau c_v}{3ne^2} \quad \propto v^2 \text{ \& } c_v$$

Classical gas: $c_v = \frac{3}{2} n k_B$
(IGL)

$$\frac{1}{2} m v^2 = \frac{3}{2} k_B T$$

$$\therefore \frac{K}{\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T \quad \propto T$$

Satisfies W-F observation: prop. const. $\approx 1.1 \times 10^{-8} \frac{W\Omega}{K^2}$

$\approx \frac{1}{2}$ exp. value.

Some LARGE oversights:

$V_c \neq V_R$ IRL: $c_v \to$ 100 x smaller

$v^2 \to$ 100 x bigger

Table. 1.6.

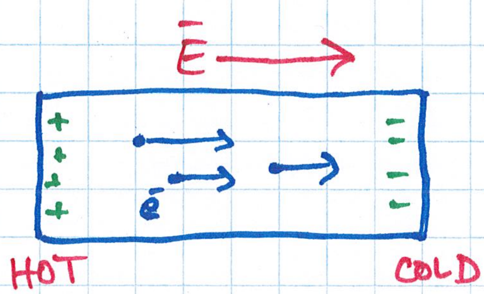
Seebeck effect: direct evidence of the Drude failure

→ a temperature gradient should be associated with an electric field in opposite direction.

– Thermoelectric effect.

$$v_H > v_c$$

→ net flow of $\bar{e}$'s from H to C until balanced by E



$$\bar{E} = Q\,\bar{\nabla} T$$

↳ thermopower (Thompson coefficient) $(V \cdot K^{-1})$

→ can be easily measured.

→ using same method as for j_q:

average v_Q →
$$\bar{v}_Q = -v\tau \frac{dv}{\partial x} = -\tau \frac{\partial}{\partial x} v^2 \qquad \text{1D}$$

$$\bar{v}_Q = -\frac{\tau}{6} \frac{dv^2}{dT} (\nabla \bar{T}) \qquad \text{3D}$$

In steady-state v_Q is balanced by $\bar{E}$ drift velocity:

$$\bar{v}_Q + \bar{v}_E = 0$$

↳ drift velocity $\bar{v}_E = -\frac{e\bar{E}\tau}{m}$

$$\therefore \frac{\tau}{6}\frac{dv^2}{dT}(\nabla \bar{T}) = -\frac{e\bar{E}\tau}{m}$$

$$\therefore \quad \bar{E} = -\frac{1}{3e}\frac{d}{dT}\frac{mv^2}{2}\,\nabla T$$

$$\therefore Q = -\frac{c_v}{3ne} \approx -0.4 \times 10^{-4}\ \frac{V}{K}$$

⟹ 100 x's too big.

DRUDE OUT!